Math 104 Final Exam Review
Spring 2009

The Final Exam will be Thursday, May 14, 4:30-7:00pm in 3139 Wescoe. Approximately two-thirds of the problems will be post-midterm material (chapters 4, 5, and 9), which is covered in this review. For review of earlier material, see the midterm review.

1. Put each equation below in standard form and determine which type of conic section it is. Depending on what type it is find the center, vertices, foci, and/or asymptotes. Use this information to sketch the graph.

   (a) $x^2 - 2x + 8y + 9 = 0$

   (b) $4x^2 + y^2 - 8x + 4y - 8 = 0$

   (c) $9x^2 - y^2 + 54x + 10y + 55 = 0$

   (d) $y^2 - 4x - 4 = 0$

   (e) $y = \frac{1}{4}(x^2 - 2x + 5)$

   (f) $9x^2 - 4y^2 + 36x - 24y + 36 = 0$

   (g) $16x^2 + 25y^2 - 32x + 50y + 16 = 0$

   (h) $y^2 + x^2 - 4x + 6y = -5$

2. Find the standard form for each of the following:

   (a) Ellipse; Vertices: (-3, 0), (7, 0); Foci: (0, 0), (4, 0)

   (b) Hyperbola; Vertices: (2, 2), (-2, 2); Foci: (4, 2), (-4, 2)

3. Sketch the curve represented by the following parametric equations (indicate the direction of the curve). Then eliminate the parameter and write the corresponding rectangular equation whose graph represents the curve

   (a) $x = 3 - 2t, y = 2 + 3t$

   (b) $x = \cos\theta, y = 3\sin\theta$

   (c) $x = \ln 2t, y = 2t^2$

   (d) $x = 3 + 3\cos\theta, y = 2 + 5\sin\theta$

   (e) $x = \frac{1}{t}, y = t^2$

   (f) $x = 6\cos\theta, y = 6\sin\theta$

   (g) $x = \sec\theta, y = \tan\theta$

   (h) $x = 1 - 2\sin\theta, y = 2 - \cos\theta$

4. For the following polar coordinates, find the corresponding rectangular coordinates.

   (a) $(4, \frac{3\pi}{2})$

   (b) $(2, -\frac{\pi}{4})$

   (c) $(2, \frac{7\pi}{6})$

   (d) $(-2, \frac{\pi}{6})$

   (e) $(1, \frac{\pi}{4})$

   (f) $(-15, 0)$

5. For the following rectangular coordinates, find two possible polar representations.

   (a) $(0, -9)$

   (b) $(-3. -\sqrt{3})$

   (c) $(2, 2)$

   (d) $(-1, 5)$

   (e) $(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$

6. Convert each rectangular equation to polar form.

   (a) $x^2 + y^2 - 6x = 0$

   (b) $4x + 7y - 2 = 0$

7. Convert each polar equation into rectangular form.

   (a) $r = 4\cos\theta$

   (b) $\theta = \frac{5\pi}{3}$

   (c) $r = 10$

   (d) $r^2 = 3\sin 2\theta$

   (e) $r\cos\theta = r^2\sin\theta$

   (f) $4\cos\theta + 2\sin\theta = r$

8. Sketch the graph of the following polar equations.

   (a) $r = 6$

   (b) $\theta = -\frac{5\pi}{3}$

   (c) $r = 6\cos\theta$

   (d) $r = 3\cos 5\theta$

9. The following parametric equations describe the flight of a baseball t seconds after being hit, where x is the horizontal distance from home plate (in feet), and y is the height (in feet):

$$x = 139.1t$$

$$y = 3 + 56.2t - 16t^2$$

   (a) What is the height of the baseball when it is hit?

   (b) What is the maximum height of the baseball?

   (c) A 10 ft. outfield fence is a distance of 400 ft from home plate. Is the hit a home run?

10. Find the amplitude, period, and vertical shift of the following and sketch a graph.

   (a) $g(x) = \cos(3x - \dfrac{\pi}{2})$

   (b) $f(\theta) = \sin(5\theta + \dfrac{5\pi}{6})$

   (c) $u(t) = 3 - \sin(t + \dfrac{\pi}{4})$

   (d) $h(x) = 2 + 5\cos(8x - 4\pi)$

11. Given $\sin\theta = -\dfrac{7}{12}$ and θ is in Quadrant IV, find the values of the other trigonometric functions.

12. The height of a radio transmission tower is 70 meters and it casts a shadow of length 30 meters. Find the angle of elevation of the sun.

13. Verify the identities.

   (a) $\sec^2 x \cot x - \cot x = \tan x$

   (b) $\cos^3 x \sin^2 x = (\sin^2 x - \sin^4 x)\cos x$

   (c) $2\sin y \cos y \sec 2y = \tan 2y$

   (d) $\dfrac{\sin(u+v)}{\cos u \cos v} = \tan u + \tan v$

14. Find all the solutions of the equation in the interval $[0, 2\pi)$.

   (a) $\sin x - \tan x = 0$

   (b) $\sin 2x + \sqrt{2}\sin x = 0$

   (c) $\cos 4x - 7\cos 2x = 8$

   (d) $\sin 4x - \sin 2x = 0$

   (e) $2\sin 2x - \sqrt{2} = 0$

   (f) $\cos 4x(\cos x - 1) = 0$

15. Use the given information to solve the triangles. If two solutions are possible, list each.

    (a) $B = 110^o, a = 4, c = 4$

    (b) $a = 5, b = 8, c = 10$

    (c) $B = 25^o, a = 6.2, C = 85^o$

16. To approximate the length of a marsh, a surveyor walked 25 meters from point A to point B. Then he turns 65^o and walks 300 meters to point C. Approximate the length AC of the marsh.

17. A boat runs in a straight line for 5 kilometers and then makes a 45^o turn and runs for another 6 kilometers. How far is the boat from its starting point?

18. The amount P (in tons) of pollution emitted by a certain factory is given by P = 3.5(1+$\cos\frac{n\pi}{26}$), where n is the number of weeks after January 1. Use inverse trigonometric notation to solve for n.

19. Find equations for two distinct ellipses both of which have center at (2, -6), major axis of length 12, and minor axis of length 8.

20. Find the exact value of the following.

    (a) $\cos(\arctan 2)$

    (b) $\cos(\arcsin\frac{4}{5})$

    (c) $\sin(\arctan(-\frac{5}{12}))$

    (d) $\tan(\arcsin(-\frac{3}{4}))$

21. A parabolic archway is 12 meters high at the vertex. At a height of 10 meters, the width of the archway is 8 meters. How wide is the archway at ground level?

22. You are building a wading pool in the shape of an ellipse. Your plans give an equation for the elliptical shape of the pool measured in feet as $\frac{x^2}{324} + \frac{y^2}{196} = 1$. Find the longest distance across the pool.

23. Find the exact value of the trigonometric functions given that $\sin u = \frac{3}{4}, \cos v = -\frac{5}{13}$, and u and v are in Quadrant II.

    (a) $\tan(u)$ (b) $\csc(v)$ (c) $\sin(2u)$ (d) $\cos(2v)$

Final Review Answers
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1. (a) Parabola; Vertex: (1, -1); Focus: (1, -3); Directrix: y = 1

   (b) Ellipse; Center: (1, -2); Vertices: (1, -6), (1, 2); Foci: (1, -2 $\pm 2\sqrt{3}$); Eccentricity: $\frac{\sqrt{3}}{2}$

   (c) Hyperbola; Center: (-3, 5); Vertices: $(-\frac{10}{3}, 5)$, $(-\frac{8}{3}, 5)$; Foci: $(-3 \pm \frac{\sqrt{10}}{3}, 5)$; Eccentricity: $\sqrt{10}$

   (d) Parabola; Vertex: (-1, 0); Focus: (0, 0); Directrix: x = -1

   (e) Parabola; Vertex: (1, 1); Focus: (1, 2); Directrix: y = 0

   (f) ; Center: (-2, -3); Vertices: (-2, -6), (-2, 0); Foci: (-2, -3$\pm\sqrt{13}$); Eccentricity: $\frac{\sqrt{13}}{3}$

   (g) Ellipse; Center: (1, -1); Vertices: $(\frac{9}{4}, -1), (-\frac{1}{4}, -1)$; Foci: $(\frac{7}{4}, -1), (\frac{1}{4}, -1)$; Eccentricity $\frac{3}{5}$

   (h) Circle: Center: (2, -3); Radius: $\sqrt{8}$

2. (a) $\frac{(x-2)^2}{25} + \frac{y^2}{21} = 1$

   (b) $\frac{x^2}{4} - \frac{(y-2)^2}{12} = 1$

3. (a) $3x + 2y - 13 = 0$

   (b) $x^2 + \frac{y^2}{9} = 1$

   (c) $y = \frac{e^{2x}}{2}$

   (d) $\frac{(x-3)^2}{9} + \frac{(y-2)^2}{25} = 1$

   (e) $y = \frac{1}{x^2}$

   (f) $x^2 + y^2 = 36$

   (g) $1 + y^2 = x^2$

   (h) $\frac{(x-1)^2}{4} + (y-2)^2 = 1$

4. (a) $(0, -4)$
   (b) $(\sqrt{2}, -\sqrt{2})$
   (c) $(-\sqrt{3}, -1)$
   (d) $(-\sqrt{3}, -1)$
   (e) $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
   (f) $(-15, 0)$

5. Answers may vary. These are some possibilities.
   (a) $(9, \frac{3\pi}{2}), (-9, \frac{\pi}{2})$
   (b) $(-2\sqrt{3}, \frac{\pi}{6}), (2\sqrt{3}, -\frac{5\pi}{6})$
   (c) $(2\sqrt{2}, \frac{\pi}{4}), (2\sqrt{2}, -\frac{-7\pi}{4})$
   (d) $(\sqrt{26}, 101.3^o), (\sqrt{26}, -258.7^o)$
   (e) $(1, \frac{4\pi}{3}), (-1, \frac{\pi}{3})$

6. (a) $r = 6\cos\theta$
   (b) $r = \dfrac{2}{4\cos\theta + 7\sin\theta}$

7. (a) $x^2 + y^2 - 4x = 0$
   (b) $x\sqrt{3} + y = 0$
   (c) $x^2 + y^2 = 100$
   (d) $(x^2 + y^2)^2 = 6xy$
   (e) $x^2 = x^2y^2 + y^4$
   (f) $x^2 - 4x + y^2 - 2y = 0$

8. Graph

9. (a) 3 feet
   (b) 52.35 feet
   (c) Yes

10. (a) Amplitude: 1; Period: $\frac{2\pi}{3}$; Phase Shift: $\frac{\pi}{6}$ right; Vertical Shift: none

    (b) Amplitude: 1; Period: $\frac{2\pi}{5}$; Phase Shift: $\frac{\pi}{6}$ left; Vertical Shift: none

    (c) Amplitude: 1; Period: 2π; Phase Shift: $\frac{\pi}{4}$ left; Vertical Shift: 3

    (d) Amplitude: 5; Period: $\frac{\pi}{4}$; Phase Shift: $\frac{\pi}{2}$ right, Vertical Shift: 2

11. (a) $\cos\theta = \frac{\sqrt{95}}{12}$

    (b) $\tan\theta = -\frac{7\sqrt{95}}{95}$

    (c) $\csc\theta = -\frac{12}{7}$

    (d) $\sec\theta = \frac{12\sqrt{95}}{95}$

    (e) $\cot\theta = -\frac{\sqrt{95}}{7}$

12. 66.8^o

13. Answers may vary

14. (a) $x = 0, \pi$

    (b) $x = 0, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}$

    (c) $x = \pi, \frac{3\pi}{2}$

    (d) $x = 0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

    (e) $x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$

    (f) $x = 0, \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

15. (a) $b \approx 6.55, A = 35^o, C = 35^o$

    (b) $A \approx 29.7^o, B \approx 52.4^o, C \approx 97.9^o$

    (c) $A = 70^o, b \approx 2.8, c \approx 6.6$

16. 311.4 meters

17. 10.2 kilometers

18. $n = \dfrac{26 \arccos(\frac{P}{3.5} - 1)}{\pi}$

19. $\dfrac{(x-2)^2}{36} + \dfrac{(y+6)^2}{16} = 1$ and $\dfrac{(x-2)^2}{16} + \dfrac{(y+6)^2}{36} = 1$

20. (a) $\dfrac{\sqrt{5}}{5}$

    (b) $\dfrac{3}{5}$

    (c) $-\dfrac{5}{13}$

    (d) $-\dfrac{3\sqrt{7}}{7}$

21. $8\sqrt{6}$ meters

22. 36 feet

23. (a) $-\dfrac{3}{\sqrt{7}}$

    (b) $\dfrac{13}{12}$

    (c) $-\dfrac{42\sqrt{7}}{112}$

    (d) $-\dfrac{119}{169}$