MATH 590 TEST NO. 4 (7/30/04)

Answer all the questions and show all your work.

1. Find the matrix of the linear transformation $f : \mathbb{R}^3 \to \mathbb{R}^3$,
   
   $$f((x_1, x_2, x_3)^T) = (x_1 + x_2 - x_3, x_2 + x_3 - x_1, -x_2)^T$$

   relative to the basis $\mathcal{B} = \{(1, 1, 0)^T, (0, 1, 1)^T, (1, 0, 1)^T\}$.

2. Use the method taught in class to solve the following system of linear differential equations:
   
   $$\begin{align*}
y_1' &= 2y_1 + 4y_2 \\
y_2' &= -y_1 - 3y_2
   \end{align*}$$

   with initial values $y_1(0) = 1, y_2(0) = 3$.

3. Let $A = \begin{pmatrix} 2 & 4 \\ -1 & -3 \end{pmatrix}$. Find a $2 \times 2$ matrix $X$ and a diagonal matrix $D$ such that $X^{-1}AX = D$.

4. Prove that for any real numbers $a, b, c, d, e$

   $$\left(\frac{a}{28} + \frac{b}{14} + \frac{c}{7} + \frac{d}{4} + \frac{e}{2}\right)^2 \leq \frac{a^2}{28} + \frac{b^2}{14} + \frac{c^2}{7} + \frac{d^2}{4} + \frac{e^2}{2}$$

5. Suppose $A$ is an $n \times n$ matrix with $n$ distinct and nonnegative eigenvalues. Prove that there is a matrix $B$ such that $B^2 = A$. 