1. Let $Z$ be the standard normal random variable. Prove that 
\[ \phi_Z(t) = e^{t^2/2}. \]

2. Prove that 
\[ \int_{-\infty}^{\infty} e^{-x^2/2} \, dx = \sqrt{2\pi}. \]

3. The life expectancy of a certain brand of light bulbs is 600 hours.
   a) Compute the probability that a randomly selected bulb lasts at least 900 hours.
   b) If one of these bulbs lasts 600 hours, what is the probability that it will last at least another 300 hours?
   c) If I install 5 of these bulbs in my kitchen, what is the probability that most of them will last at least 600 hours?

4. The weekly rainfall in a certain equatorial city is normally distributed with a mean of 10.8” and a variance of 3.5”.
   a) Estimate the probability that the total rainfall over the next year (52 weeks) will not exceed 550”.
   b) A week is considered to be dry if its total rainfall is less than 4”. What is the probability that none of the next four weeks are will be dry?

5. 20% of the oranges of a certain shipment are spoiled.
   a) If 12 of these oranges are selected at random, compute the probability that between 1 and 3 are spoiled (Do not use the normal approximation here.)
   b) If 40 of these oranges are selected at random, estimate the probability that between 7 and 9 are spoiled.