MATH 661 TEST NO. 2 (4/17/06)

This is an open book test. Answer all the questions and justify your answers.

1. Let $c > 0$. Prove that the transformation $(\tilde{u}, \tilde{v}) = f(u, v) = (cu, cv)$ is an isometry of $\mathcal{H}$ in the upper half-plane.

2. Compute the angle between the lines $u = 0$ and $v = 1$ relative to the metric
   $$\frac{2du^2 + 3dv^2}{v^2}.$$

3. Compute the curvature of the metric
   $$\frac{du^2 + dv^2}{v^3}$$
in the upper half-plane.

4. Let $ABC$ be a hyperbolic right triangle with right angle at $C$. Prove that
   $$\tanh b = \sinh a \tan \beta$$

5. Compute the half-plane area of the region $R$ in the figure below, where all three arcs are Euclidean semicircles centered on the $x$-axis.