Show all your work and justify your answers. You may use your calculators.

1. Let $X$ be a Poisson random variable. Prove that $E[X] = Var(X) = \lambda$.

2. Suppose 300 chips are used to make 60 chocolate chip cookies.
   a) Estimate the number of cookies that contain at least 4 chips.
   b) Five of the cookies are selected at random. What is the probability that at least 3 of them contain at most 4 chips each?

3. A fair coin is flipped three times. $X$ denotes the total number of heads whereas $Y$ denotes the number of heads on the first flip. Construct the joint distribution table of $X$ and $Y$. Compute the marginal probability densities and use them to decided whether $X$ and $Y$ are independent or not.

4. When flipped, an unfair coin is twice as likely to come up heads as tails. If the coin is flipped 24 times, compute the probabilities of the following events.
   a) The number of tails is between 22 and 26 (inclusive).
   b) Exactly twice as many heads show up as tails.

5. a) A number is selected at random from the set $\{1, 2, \ldots, 10\}$. Are divisibility by 2 and by 3 independent events?
   b) A hand of six cards is dealt at random from a standard deck. What is the probability that the hand contains cards from all four suits?