MATH 646 Test No. 1
October 8, 2004

Answer all the questions, justify your answers and show your work.

1. Prove one of the following two propositions:
   A. Suppose that the function \( f(z) = u + iv \) has a derivative at \( z_0 \). Prove that at that point we have \( u_x = v_y \), and \( u_y = -v_x \).
   B. Suppose \( f : D \to \mathbb{C} \) is such that \( \int_{\gamma} f(z)dz \) is independent of path. Prove that \( f \) has an antiderivative in \( D \).

2. Is the equation below valid?
   \[
   \int_{C_1(0)} \log z \, dz = \int_{C_1(0)} \log (z + 2) \, dz
   \]

3. Find the principal value of \((-i)(-i)\).

4. Explain why \( \lim_{z \to 0} \frac{\text{Im}(z)}{z} \) does not exist.

5. Let \( f(z) = u + iv \) be an analytic function such that \( uv = 1 \). Prove that \( f \) is a constant.