Answer all the questions, justify your answers and show your work.

1. Prove that $a^{560} \equiv 1 \pmod{561}$ for every $a$ that is relatively prime to 561.

2. What is the remainder of 53! when divided by 59?

3. Is the function $n\phi(n)$ is multiplicative? Evaluate both its ”derivative” and ”integral” for $n = 35, 36, 37$.

4. For a certain RSA public key encryption, the number $m$ is encrypted as $m^{81} \pmod{2101}$. Break this code.

5. Let $n$ be a positive integer such that $\phi(n)|(n−1)$. Prove that $\mu(n) \neq 0$. 