MATH 540 Test No. 1
October 11, 2005

Answer all the questions, justify your answers and show your work.

1) Solve the Diophantine equations $91x + 221y = 1053$ and $91x + 221 = 1054$.

2) Prove that for integers $3 \leq k \leq n$:

$$\binom{3}{3} + \binom{4}{3} + \binom{5}{3} + \ldots + \binom{n}{3} = \binom{n + 1}{4}$$

3) For which primes $p$ is $7p + 4$ a perfect square?

4) Solve the congruence $11x \equiv 28 \pmod{1943}$

5) Compute the remainder of $56!$ when divided by 59.

6) Let $p$ and $q$ be distinct odd primes such that $p - 1$ divides $q - 1$. Prove that if $(a, pq) = 1$ then $a^{q-1} \equiv 1 \pmod{pq}$. 