

Math Awareness Week Competition
2007 Examination for 10th-12th Grades

DIRECTIONS: [40 Minutes - 5 Questions] Start each new problem on a separate page. **Show your work!** Answers must be exact. You are allowed to use a calculator. You are not allowed to borrow or interchange calculators during the test.

1. A biologist wants to calculate the number of fish in a lake. On May 1, she catches a random sample of 60 fish, tags them and releases them. On September 1, she catches a random sample of 70 fish and finds that 3 of them are tagged. To calculate the number of fish in the lake on May 1, she assumes that 25% of these fish are no longer in the lake on September 1 (because of death and emigrations), and that 40% of the fish present on September 1 were not in the lake on May 1 (because of births and immigrations) and the number of untagged fish and the tagged fish in the September 1 sample are representative of the total population. Find the number of fish in the lake on May 1 as calculated by the biologist.
2. Four positive integers, when added three at a time, give the sum 180, 197, 208, and 222. Find the largest of the four numbers?
3. Consider the finite sequence $\langle a_1, a_2, \dots, a_n \rangle$ of real numbers. The Cesàro sum of A is

$$\frac{S_1 + S_2 + \dots + S_n}{n}$$

where $S_k = a_1 + a_2 + \dots + a_k$ for $1 \leq k \leq n$. If the Cesàro sum of the 99-term sequence $\langle a_1, a_2, \dots, a_{99} \rangle$ is 1000, find the Cesàro sum of the 100-term sequence $\langle 1, a_1, a_2, \dots, a_n \rangle$.

4. If a , b , and c are three (not necessarily different) numbers chosen randomly and with replacement from the set $\{1, 2, 3, 4, 5\}$, find the probability that $ab + c$ is even.
5. In the figure, $ABCD$ is a quadrilateral with right angles at A and C . Points E and F are on \overline{AC} , and \overline{DE} , and \overline{BF} are perpendicular to \overline{AC} . If $AE = 3$, $DE = 5$ and $CD = 7$, find BF .

