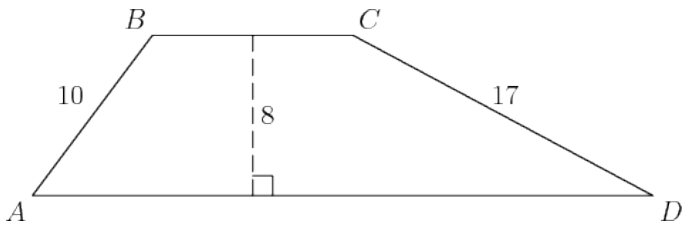
Math Awareness Month Competition
2013 Examination for 6th-8th Grades

DIRECTIONS: [40 Minutes - 5 Questions] Start each new problem on a separate page. **Show your work!** Answers must be exact. You are allowed to use a calculator. You are not allowed to borrow or interchange calculators during the test.

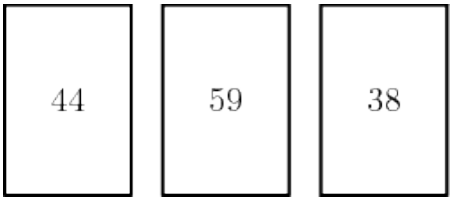
1. Each of the 39 students in the seventh grade at Jayhawk Middle School has one dog or one cat or both a dog and a cat. Twenty students have a dog and 26 students have a cat. How many students have both a dog and a cat?

2. The area of trapezoid ABCD is 164 cm^2. The altitude is 8 cm, AB is 10 cm, and CD is 17 cm. What is BC, in centimeters?



3. Before district play, the basketball team won 45 of their basketball games. During district play, they won six more games and lost two, to finish the season having won half their games. How many games did the team play in all?

4. Barry wrote 6 different numbers, one on each side of 3 cards, and laid the cards on a table, as shown. The sums of the two numbers on each of the three cards are equal. The three numbers on the hidden sides are prime numbers. What are the hidden prime numbers?



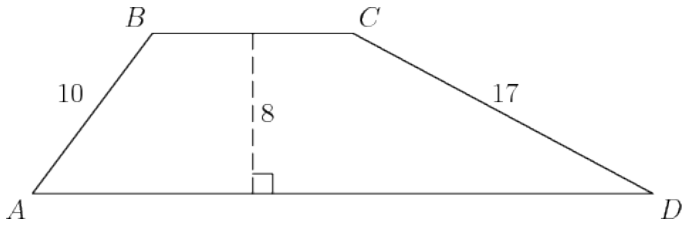
5. Ten tiles numbered 1 through 10 are turned face down. One tile is turned up at random, and a die is rolled. What is the probability that the product of the numbers on the tile and the die will be a square?

Answers:

1. Each of the 39 students in the seventh grade at Jayhawk Middle School has one dog or one cat or both a dog and a cat. Twenty students have a dog and 26 students have a cat. How many students have both a dog and a cat?

   [Solution: The union of two sets is equal to the sum of each set minus their intersection. The number of students that have both a dog and a cat is 20+26-39 = 7.]

2. The area of trapezoid ABCD is 164 cm^2. The altitude is 8 cm, AB is 10 cm, and CD is 17 cm. What is BC, in centimeters?

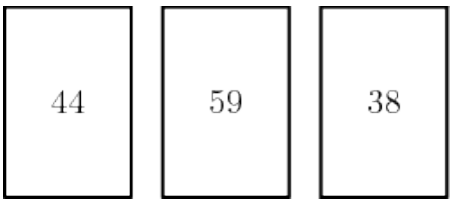


   [Solution: Using the formula for the area of a trapezoid, we have $164 = 8(\frac{BC+AD}{2})$. Thus $BC + AD = 41$. Drop perpendiculars from B to AD and from C to AD and let them hit AD at E and F respectively. Note that each of these perpendiculars has length 8. From the Pythagorean Theorem, $AE = 6$ and $DF = 15$ thus $AD = BC + 21$. Substituting back into our original equation we have $BC + BC + 21 = 41$, thus $BC = 10$.]

3. Before district play, the basketball team won 45 of their basketball games. During district play, they won six more games and lost two, to finish the season having won half their games. How many games did the team play in all?

   [Solution: Before the district play, the team won 45 games. During the district play, they won 6 more games. They finished the season with a total win of 51 games. If this is half of their total games, they played $2 \cdot 56 = 102$ games.]

4. Barry wrote 6 different numbers, one on each side of 3 cards, and laid the cards on a table, as shown. The sums of the two numbers on each of the three cards are equal. The three numbers on the hidden sides are prime numbers. What are the hidden prime numbers?



[Solution: Notice that 44 and 38 are both even, while 59 is odd. If any odd prime is added to 59, an even number will be obtained. However, the only way to obtain this even number would be to add another even number to 44, and a different one to 38. Since there is only one even prime (2), the middle card's hidden number cannot be an odd prime, and so must be even. Therefore, the middle card's hidden number must be 2, so the constant sum is 61. Thus, the first card's hidden number is 61-44 = 17, and the last card's hidden number is 61-38 = 23.]

5. Ten tiles numbered 1 through 10 are turned face down. One tile is turned up at random, and a die is rolled. What is the probability that the product of the numbers on the tile and the die will be a square?

[The numbers can at most multiply to be 60. The squares less than 60 are1,4,9,16, 25,36, and 49. The possible pairs are (1,1), (1,4),(2,2),(4,1), (3,3), (9,1), (4,4), (8,2), (5,5), (6,6), and (9,4). There are 11 choices and 60 possibilities giving a probability of $\frac{11}{60}$.]