

## Math Awareness Month Competition 2012 Examination for 10-12th Grades

---

**DIRECTIONS:** [40 Minutes - 5 Questions] Start each new problem on a separate page. **Show your work!** Answers must be exact. You are allowed to use a calculator. You are not allowed to borrow or interchange calculators during the test.

---

1. Alice and Bob are attending a dinner with 10 other people. All people are seated randomly in a row. What is the probability that Alice and Bob will seat next to each other?
2. Let  $X_1, \dots, X_{10}$  be 10 consecutive integers. Find the last digit of

$$\sum_{i=1}^{10} X_i^{17}$$

3. Find the maximal value of  $\cos^{2012}(x) - \sin^{2013}(x)$ .
4. In David's math class, the following game is played: the teacher writes three numbers,  $A, B$  and  $C$ , between 0 and 2012 on the board. Each time, a student erases all the numbers  $A, B, C$  on the board and replaces them with  $\frac{A+B}{2}, \frac{B+C}{2}, \frac{A+C}{2}$ . After 11 students finish their turns, there are three integers on the board, the smallest of which is 100. What is the largest number on the board at that point?
5. Consider a square  $ABCD$  whose sides are equal to 1. Let  $E, F$  be two points on the sides  $BC, CD$  respectively. The lines  $AE, AF$  intersect the diagonal  $BD$  at  $G, H$  respectively. Assume that the length of  $GH$  is exactly half of the length of  $BD$ . What is the minimal value of  $BE + DF$ ?