

## Math Awareness Month Competition 2011

### Solutions for 7th-9th Grades

1. A two-digit number  $ab$  is called “good” if  $a^2 + b^2 = 65$ . How many good numbers are there?

[Solution: There are 4 numbers: 81, 18, 74, 47.]

2. Amy, Bob and Celine deliver papers in a small town. Amy delivers twice as many papers as Bob, and Carrie delivers 12 papers less than Amy. If Amy, Bob, and Celine deliver 128 papers in total, how many papers does Carrie deliver?

[Solution: Let  $x$  = the number of papers Bob delivers. Then Amy delivers  $2x$  papers, and Carrie delivers  $2x - 12$  papers. Thus  $x + 2x + 2x - 12 = 128$  and  $x = 140/5 = 28$ . The final answer is  $2 \times 28 - 12 = 44$ .]

3. Find the radius of the circle inscribed inside the right triangle  $ABC$  with  $\overline{AB} = 3$ ,  $\overline{BC} = 4$ ,  $\overline{CA} = 5$ .

[Solution: Let  $O$  be the center and  $r$  be the radius. Then the area of  $OAB$  is

$$\text{Area}(OAB) = (1/2)r\overline{AB}$$

and similarly for  $OBC, OCA$ . Adding up we have:

$$2\text{Area}(ABC) = r(\overline{AB} + \overline{BC} + \overline{CA}) = 12r$$

But twice the area of  $ABC$  is also  $3 \times 4 = 12$ . So  $r = 1$ .]

4. How many digits does  $2^{100}$  have?

[Solution: The answer is 31. Note that  $10^{30} < 2^{100} < 10^{31}$ . The first inequality follows because  $2^{10} = 1024 > 10^3$ . The second follows because  $2^{13} = 1024 \times 8 < 10^4$ , so

$$2^{100} = 2^{91}2^9 < 10^{4 \times 7} \times 10^3 = 10^{31}]$$

5. A  $5 \times 5$  grid is divided into 25 small  $1 \times 1$  squares. Each small square has a frog inside. After a whistle, each frog jumps to an adjacent square. What is the least possible number of squares with at least two frogs in it?

[Solution: The answer is ONE. Color the grid alternatively with black and white, with the top left corner square black. There are 13 black and 12 white squares. After the whistle, each frog in a black square (there are 13 of them) is now in a white square. So one white square must have at least two frogs in it. It is easy to construct a way of jumping resulting in only one such square: just pick a path through all the squares once, and make the frogs jump to the next one in the path, with the last one jumps to any adjacent square.]