1. Solve the equation: \( \log_2 x + (x - 1) \log_2 x = 6 - 2x \).

[Solution: Let \( y = \log_2 x \); so, \( x = 2^y \). The equation becomes \( y^2 - (2^y - 1)y = 6 - 2 \cdot 2^y \). That is, we have: \( 0 = y^2 - (2^y - 1)y - 6 + 2 \cdot 2^y = (y + 2)(y + 2^y - 3) \). So, \( y = -2 \) and \( x = \frac{1}{4} \) or \( y + 2^y - 3 = 0 \) and \( y = 1, x = 2 \). The solutions are \( x = \frac{1}{4} \) and \( x = 2 \).]

2. The sides of the polygon \( ABCD \) are given in centimeters as follows: \( AB = 8 \) cm, \( BC = 16 \) cm, \( CD = 4 \) cm, and \( AD \) cm. What is the length of the diagonal \( BD \) if it is known that it is an integer number in centimeters.

[Solution:

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From \( \triangle ABD \), \( BD < 6 + 8 = 14 \). From \( \triangle BCD \), \( BD > 16 - 4 = 12 \). So, \( BD = 14 \) centimeters.]
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3. Solve the equation: \( (2 \sin x - 1)(\sin x + 1) = \cos x \).

[Solution: From \( \cos x = 2 \sin^2 x + 2 \sin x - \sin x - 1 = 2 \sin^2 x + \sin x - (\sin^2 x + \cos^2 x) \), we get \( \sin^2 x - \cos^2 x = \cos x - \sin x \) and \( (\sin x + \cos x)(\sin x - \cos x) = -(\sin x + \cos x) \). So, \( \sin x - \cos x = 0 \) or \( \sin x + \cos x = 0 \). The solutions are \( x = \frac{\pi}{4} + k\pi \) and \( x = -\frac{\pi}{2} + 2k\pi \) where \( k \) is an integer.]

4. The bases for a trapezoid \( ABCD \) are \( AB = 9 \) and \( CD = 3 \) and the other two sides are \( AD \) and \( BC \). \( M \) is a point on the side \( BC \) and \( N \) is a point on the side of \( AD \) such that \( MN \) is parallel to \( AB \) and \( CD \) and \( MN = 7 \). Find the quotient of the area of \( ABMN \) and the area of \( NMCD \), i.e., compute

\[
\frac{S_{ABMN}}{S_{NMCD}} = ?
\]
Let $T$ be the point of intersection of $AD$ and $BC$. Now, $\frac{S_{\triangle ABT}}{S_{\triangle NMT}} = \frac{81}{49}$ and $\frac{S_{\triangle NMT}}{S_{\triangle DCT}} = \frac{49}{9}$.

\[
\frac{S_{ABMN}}{S_{NMC}} = \frac{S_{\triangle ABT} - S_{\triangle NMT}}{S_{\triangle NMT} - S_{\triangle DCT}} = \frac{\frac{81}{49}S_{\triangle NMT} - S_{\triangle NMT}}{S_{\triangle NMT} - \frac{9}{49}S_{\triangle NMT}} = \frac{49}{40} = \frac{4}{5}.
\]

5. If the equation
\[x^4 - ax^3 + 2x^2 - bx + 1 = 0\]
has a real root, show that
\[a^2 + b^2 \geq 8.
\]

[Solution: Let $y$ be a real root of the equation. Then $0 = y^4 - ay^3 + 2y^2 - by + 1$, and
\[0 = y^2 - ay + 2 - b\frac{1}{y} + 1\frac{1}{y^2}.
\] Completing the square, we get
\[(y - \frac{a}{2})^2 + \left(\frac{1}{y} - \frac{b}{2}\right)^2 = \frac{a^2 + b^2 - 8}{4} \geq 0.
\] Thus, $a^2 + b^2 \geq 8$.]

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