Math Awareness Month Competition

2008 Examination for 7th-9th Grades

**DIRECTIONS:** [40 Minutes - 5 Questions] Start each new problem on a separate page. **Show your work!** Answers must be **exact.** You are allowed to use a calculator. You are not allowed to borrow or interchange calculators during the test.

1. How many four-digit whole numbers are there such that the leftmost digit is odd, the second digit is even, and all four digits are different?

2. An isosceles right triangle $ABC$ encloses a semicircle of area $2\pi$. The circle has its center $O$ on hypotenuse $AB$ and is tangent to the sides $AC$ and $BC$. Find the area of triangle $ABC$.

3. A pair of 8-sided dice have sides numbered 1 through 8. Each side has the same probability of landing face up. Find the probability that the product of the two numbers on the sides that land face-up exceeds 36.

4. Andy’s lawn has twice as much area as Beth’s lawn and three times as much area as Carlos’ lawn. Carlos’s lawn mower cuts half as fast as Beth’s mower and one third as fast as Andy’s mower. It takes Carlos 60 minutes to mow his lawn. How long does it take for Andy to mow his lawn and how long does it take for Beth to mow her lawn.

5. A subset $A$ of the set of integers from 1 to 100, inclusive, has the property that no two elements of $A$ sum to 125. What is the maximum possible number of elements of $A$?