Math Awareness Month Competition

2008 Examination for 10th-12th Grades

**DIRECTIONS:** [40 Minutes - 5 Questions] Start each new problem on a separate page. **Show your work!** Answers must be **exact.** You are allowed to use a calculator. You are not allowed to borrow or interchange calculators during the test.

1. There exist positive integers $A$, $B$, and $C$, with no common factor greater than 1, such that

$$A \log_{200} 5 + B \log_{200} 2 = C.$$ 

Find $A$, $B$, and $C$.

2. A *rising* number, such as 34689, is a positive integer each digit of which is larger than each of the digits to its left. There are \( \binom{9}{5} = 126 \) five-digit rising numbers. When these numbers are arranged from smallest to largest, find the 97th number in the list.

3. Six distinct integers are picked at random from \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}. What is the probability that among those selected, the second smallest is 3?

4. In \( \triangle ABC \), \( \angle ABC = 120^\circ \), \( AB = 3 \) and \( BC = 4 \). If perpendiculars constructed to \( AB \) at \( A \) and to \( BC \) at \( C \) meet at \( D \), find \( CD \).

5. A sequence of complex numbers \( z_0, z_1, z_2, \cdots \) is defined by the rule \( z_{n+1} = \frac{iz_n}{\bar{z}_n} \) where \( \bar{z}_n \) is the complex conjugate of \( z_n \) and \( i^2 = -1 \). Suppose \( |z_0| = 1 \) and \( z_{10} = 1 \). How many possible values are there for \( z_0 \)?