Directions: Work each problem on a separate sheet of paper. Each problem is worth the same number of points.

1. A topological space $X$ has the fixed point property (fpp) if for any continuous function $f : X \to X$, there is an $x_0$ with $f(x_0) = x_0$. Prove that:
   - The disk $D := \{(x,y) : x^2 + y^2 \leq 1\}$ has the fpp. (You may assume that $\pi(S^1)$ is infinite cyclic.)
   - If $X$ has the fpp and $Y$ is a retract of $X$, then $Y$ has the fpp.

2. Define the term covering space. Let $(\tilde{X}, \tilde{p})$ be a covering space of the arcwise connected space $X$. Prove that the cardinality of $\tilde{p}^{-1}(x)$ is constant for $x \in X$. You may assume unique path lifting.

3. State the Seifert Van Kampen theorem. Use it (and/or its corollaries) to compute the fundamental group of the space below: Be sure to justify (briefly) your work.

4. In the homology sequence of the pair $(X, A)$:

$$\cdots \to H_n(A) \xrightarrow{i_*} H_n(X) \xrightarrow{j_*} H_n(X, A) \xrightarrow{\partial_*} H_{n-1}(A) \to \cdots$$

carefully define $\partial_*$. Show that $\partial_*$ is well-defined on homology classes.
5. Use induction and the Mayer-Vietoris sequence to compute $\bar{H}_q(S^n)$.

6. Prove that there exists a nowhere vanishing vectorfield on $S^n$ if and only if $n$ is odd.