Math 122 Midterm Exam, March 11, 2003

Name: Instructor:

Directions: This is a closed-book test with no calculators. You MUST show all your work to receive credit. Each problem is worth 20 points out of a total of 200.

1. Determine if the following series converge or diverge. You must justify your answer (e.g., “the series converges by comparison with . . . ”).
   a. \( \sum_{n=1}^{\infty} \frac{1}{n^2 + n} \)
   b. \( \sum_{n=1}^{\infty} \frac{\ln(n)}{n} \)
   c. \( \sum_{n=0}^{\infty} \frac{6^n}{n!} \)
   d. \( \sum_{n=0}^{\infty} (-1)^n \left( \frac{3}{2} \right)^{n-1} \)

2. Find the radius of convergence \( \rho \) of the power series \( \sum_{n=1}^{\infty} \frac{4^n}{\sqrt{n}} x^n \).
   Determine whether or not the series converges at \( \pm \rho \); again, do not forget to justify your answers.

3. Starting with the power series for \( e^x \), find a power series expression for \( \int x^3 e^x \, dx \).

4. Find the Taylor series for \( f(x) = e^x \) centered at \( a = 1 \). Find the interval of convergence of the series.

5. What is the Maclaurin series for \( f(x) = \sin(x) \)? Find an approximate value for \( f(2) \) by using the first three non-zero terms of this series.

6. Find the parametric equations for the line of intersection of the two planes \( 2x - y + z = 4 \) and \( -x + y + z = 6 \).

7. Find the equation of the plane which contains the point \((2, -3, 3)\) and is perpendicular to the line
   \[ \frac{x-1}{2} = \frac{y+1}{4} = \frac{z}{3} \]

8. Define the angle between two planes to be the angle between their respective normal vectors. If \( \theta \) is the angle between the two planes \( 2x - y + z = 1 \) and \( x + y + z = 10 \), find an expression for \( \cos(\theta) \).

9. Find the area bounded by the polar curve \( r = 2 \sin(\theta) \) in the upper half plane \( (0 \leq \theta \leq \pi) \).

10. a. Convert the equation \( r = \cos(\theta) - \sin(\theta) \) into one in Cartesian (rectangular) coordinates.
    b. Find the center and radius of the sphere \( x^2 - 2x = 6y - y^2 - z^2 + 26 \).