

Math Awareness Month Competition
University of Kansas, Department of Mathematics
2017 Examination for 3rd-5th Grades

DIRECTIONS: You have 40 minutes for the five problems. Show all of the necessary work and provide a complete justification for each answer. Enclose each answer in a box. Solve each problem on a separate sheet of paper. You are allowed to use a calculator but you are not allowed to borrow or interchange calculators during the test.

1. What numbers should replace the symbols such that the calculations agree?

$$\text{KU} = ? \quad \star = ? \quad \clubsuit = ? \quad \spadesuit = ? \quad \blacklozenge = ?$$

$$\begin{array}{cccccc} \text{KU} & + & \text{KU} & - & \text{KU} & = & \text{KU} \\ + & & + & & + & & + \\ \star & - & 20 & + & \star & = & \star \\ + & & - & & + & & + \\ \clubsuit & + & \clubsuit & + & \clubsuit & = & 90 \\ = & & = & & = & & = \\ 60 & + & \spadesuit & + & \blacklozenge & = & 120 \end{array}$$

Solution: $\text{KU} = 10 \quad \star = 20 \quad \clubsuit = 30 \quad \spadesuit = 0 \quad \blacklozenge = 60$

2. What will be the next number in the sequence and why?

$$4 \quad 11 \quad 25 \quad 53 \quad \dots$$

Solution: The next number in the sequence is **109**, because $a_n = 2a_{n-1} + 3$.

3. There are two twin brothers at a crossroad. One of them always tells the truth and the other always lies. One of the roads leads to the city, and the other to the swamp. You meet one of the brothers (you do not know which one) and you can ask him only one question. What question should you ask if you want to get to the city?

Solution: Which way to the city would your brother point to me? After such a question, each of the brothers will point the way to the swamp.

4. Each time Jay fills out a form, he marks just one box: A, B, or C. If he checks boxes at random, what is the probability that in filling out 3 such forms he will mark one each of A, B, and C?

Solution: Whatever box Jay checks 1st, the probabilities are $2/3$ that he checks a different one on the 2nd form and $1/3$ that the 3rd form differs from the first two. So $2/3 \times 1/3 = 2/9$.

5. The numbers $1, \dots, 9$ are written on separate cards. Players alternately take one of the cards. The player who first completes three cards with a total sum of 15 wins. The starting player has selected the card with 2. What should the second player do?

Hint: To oversee the possible outcomes, organize the cards in a “magic square”. It has three columns and three rows, and the sum of each row and each column and each diagonal is 15.

Solution: Second player should take the number 5, otherwise he will loose.

The magic square is

2	7	6
9	5	1
4	3	8

A player wins if he completes a row, column, or diagonal. If the starting player chooses the number 2, that is, he will take the corner field, he can always win unless the second player immediately takes the middle field (that is, take the number 5).