Math Awareness Month Competition
2013 Examination for 3rd-5th Grades

DIRECTIONS: [40 Minutes - 5 Questions] Start each new problem on a separate page. **Show your work!** Answers must be exact. You are allowed to use a calculator. You are not allowed to borrow or interchange calculators during the test.

1. Two different circles overlap. The area of the overlapping region is 1/2 of the area of the smaller circle and is 1/6 of the sum of the area of the larger circle plus the area of the smaller circle. What is the ratio of the area of the smaller circle to the larger circle?

2. James needs 51 cents to buy a chocolate bar, while Steven needs 45 cents to buy it. Together they can buy one chocolate bar with 2 cents remaining. How much money does Steven have?

3. If we write all of the whole numbers from 10 through 99 and cross out any number in which the first or second digit is divisible by 3 (for example, 20 is crossed out as 0 is divisible by 3 and 91 is crossed out as 9 is divisible by 3), how many numbers are crossed out?

4. Four girls competed in a running contest and placed first, second, third, and fourth. After the contest, each girl is asked where she placed. Fran says she placed neither in first nor fourth. Gabby says she did not place in first. Heidi says she placed in first. Iris says she placed in fourth. A reliable witness says three of these answers are true and one is false. In what position did Gabby placed?

5. Erik has a bag of 58 balls, some are red and the rest are white. He removes some of the balls from the bag and notices that he removed 6 times more white balls than red. He observes that of the balls remaining in the bag, there are 5 times more red balls than white. How many balls did he remove from the bag and how many were red?

1. Two different circles overlap. The area of the overlapping region is 1/2 of the area of the smaller circle and is 1/6 of the sum of the area of the larger circle plus the area of the smaller circle. What is the ratio of the area of the smaller circle to the larger circle?
   [Solution: Overlapped area = Small's area /2 = (Large's area + Small's area) /6. So Small's area / Large's area = 1:2.]

2. James needs 51 cents to buy a chocolate bar, while Steven needs 45 cents to buy it. Together they can buy one chocolate bar with 2 cents remaining. How much money does Steven have?
   [Solution: If they team up and buy a chocolate bar, $51 + 45 + 2 = 98$ cents are missing to buy another one. So a chocolate bar costs 98 cents, and Steven has $98 - 45 = 53$ cents.]

3. If we write all of the whole numbers from 10 through 99 and cross out any number in which the first or second digit is divisible by 3 (for example, 20 is crossed out as 0 is divisible by 3 and 91 is crossed out as 9 is divisible by 3), how many numbers are crossed out?
   [Solution: The digits divisible by 3 are 0, 3, 6, 9. If the first digit is 3, 6, or 9, then the second digit can be 0,1,2,...,9. If the second digit is one of the four digits, 0, 3, 6, 9, then the first digit can be 1,2,3,...,9. So the total number of times is $3 * 10 + 4 * 6 = 54$.]

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   [Solution: Heidi is right, she is the first, otherwise the other three answers would be correct and nobody would be the first. Then Gaby is correct, too. Fran must be also right, otherwise she would be the last (because could not be the first) but then Iris would be wrong. So only Iris can be wrong, and this is possible if Gaby is the last, Fran is the second and Iris is the third.]

5. Erik has a bag of 58 balls, some are red and the rest are white. He removes some of the balls from the bag and notices that he removed 6 times more white balls than red. He observes that of the balls remaining in the bag, there are 5 times more red balls than white. How many balls did he remove from the bag and how many were red?
   [Solution: The number of balls Erik took out is a multiple of 7, and the number of balls remained in the bag is a multiple of 6. Hence 58-7, 58-14,... must be divisible by 6. Only $58 - 28 = 30$ satisfies that, so he took out 28 balls, 4 red and 24 white. In the remaining 30, there were 5 white and 25 red. So originally, there were 29 white and 29 red.]