

Math Awareness Month Competition

2009 Examination for 10th-12th Grades

DIRECTIONS: [40 Minutes - 5 Questions] Start each new problem on a separate page. **Show your work!** Answers must be **exact**. You are allowed to use a calculator. You are not allowed to borrow or interchange calculators during the test.

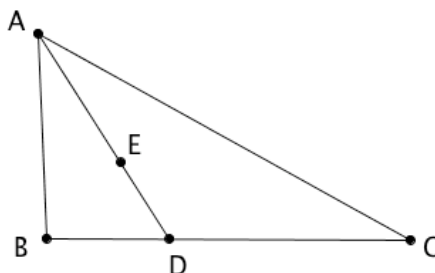
1. For each positive integer n , define $f(n) = \begin{cases} \log_8 n & \log_8 n \text{ is rational} \\ 0 & \text{otherwise.} \end{cases}$

Compute $\sum_1^{2009} f(n)$.

2. In the magic square shown, the sums of the numbers in each row, column, and diagonal are the same. Find x .

x	19	96
1		

3. Triangle ABC is a right triangle with $\angle ABC$ a right angle, $m\angle BAC = 60^\circ$, and $AC = 10$. Let E be a randomly chosen point in $\triangle ABC$, and extend \overline{AE} to meet \overline{BC} at D . Find the probability that $AD > 5\sqrt{2}$.



4. The function f has the property that for real number x in its domain, $\frac{1}{x}$ is also in its domain and $x = f(x) + f\left(\frac{1}{x}\right)$. Find the largest set of real numbers that can be in its domain of f .
5. What is the number of ordered pairs of real numbers (a, b) such that $(a + bi)^{2009} = a - bi$?