

Math Awareness Month Competition

2008 Examination for 10th-12th Grades

DIRECTIONS: [40 Minutes - 5 Questions] Start each new problem on a separate page. **Show your work!** Answers must be **exact**. You are allowed to use a calculator. You are not allowed to borrow or interchange calculators during the test.

1. There exist positive integers A , B , and C , with no common factor greater than 1, such that

$$A \log_{200} 5 + B \log_{200} 2 = C.$$

Find A , B , and C .

2. A *rising* number, such as 34689, is a positive integer each digit of which is larger than each of the digits to its left. There are $\binom{9}{5} = 126$ five-digit rising numbers. When these numbers are arranged from smallest to largest, find the 97th number in the list.

3. Six distinct integers are picked at random from $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. What is the probability that among those selected, the second smallest is 3?

4. In $\triangle ABC$, $\angle ABC = 120^\circ$, $AB = 3$ and $BC = 4$. If perpendiculars constructed to \overline{AB} at A and to \overline{BC} at C meet at D , find CD .

5. A sequence of complex numbers z_0, z_1, z_2, \dots is defined by the rule $z_{n+1} = \frac{iz_n}{\overline{z_n}}$ where $\overline{z_n}$ is the complex conjugate of z_n and $i^2 = -1$. Suppose $|z_0| = 1$ and $z_{10} = 1$. How many possible values are there for z_0 ?